Soliton stability in a bimodal optical fiber in the presence of the Raman effect

Boris A. Malomed*

Raymond and Beverly Sackler Faculty of Exact Sciences, Department of Applied Mathematics, School of Mathematical Sciences, Tel Aviv University, Ramat Aviv 69978, Israel

(Received 15 June 1994)

Using the quasi-instantaneous approximation for the Raman interactions in the bimodal birefringent optical fiber, and a simple equal-width approximation for a two-component soliton, it is demonstrated that, in virtue of the known constraint imposed by the isotropy of nonlinearity and reality of polarizability, single-component solitons are stable, while symmetric vector solitons are not.

PACS number(s): 42.81.Dp, 42.81.Gs

The important impact of interaction between different polarizations on propagation of light in nonlinear optical fibers is well known (see, e.g., Ref. [1]). It is also well known that stimulated Raman scattering may essentially affect dynamics of ultrashort solitons in the fibers [2]. Recently, it was considered in detail how these two effects combine, i.e., the Raman downshift involving two polarized components of light in a bimodal nonlinear fiber [3,4]. In the quasi-instantaneous approximation, i.e., when the finite delay of the Raman response is neglected, the coupled nonlinear Schrödinger (NLS) equations incorporating the Raman terms take the form [3]

$$iu_z + i\delta u_\tau + \tfrac{1}{2}u_{\tau\tau} + (|u|^2 + \tfrac{2}{3}|v|^2)u = \epsilon_1\left(|u|^2\right)_\tau u + \epsilon_2\left(|v|^2\right)_\tau u + \epsilon_3\left(uv^*\right)_\tau v\,, \quad (1)$$

$$iv_z - i\delta v_\tau + \tfrac{1}{2}v_{\tau\tau} + (|v|^2 + \tfrac{2}{3}|v|^2)v = \epsilon_1\left(|v|^2\right)_\tau v + \epsilon_2\left(|u|^2\right)_\tau v + \epsilon_3\left(vu^*\right)_\tau u\,, \quad (2)$$

where u and v are envelopes of two linearly polarized components of the electromagnetic wave, z is the propagation distance, t is time, and $\tau \equiv t - z/V_{\rm gr}$, $V_{\rm gr}$ being the mean group velocity of the light in the fiber. The coefficient δ measures the birefringence-induced group velocity difference between the polarizations, ϵ_1 is the coefficient of the *parallel* Raman effect, and ϵ_2 and ϵ_3 are the so-called *perpendicular* Raman coefficients (only ϵ_3 contributes to the perpendicular Raman gain). These coefficients obey a fundamental relation following from the isotropy of nonlinearity and reality of polarizability of the optical medium [3]:

$$\epsilon_1 = \epsilon_2 + 2\epsilon_3\,. \quad (3)$$

The quasi-instantaneous form of the Raman terms in Eqs. (1) and (2) is but a simplest approximation, while a more accurate model should include the full time-delayed Raman response [3,4]. Accordingly, Eq. (3) is a special case of a general relation derived in Ref. [3] for coefficient functions describing the time-delayed response.

One can exclude the birefringence terms from Eqs. (1) and (2) by means of the transformation $u(z,\tau) \equiv U(z,\tau)\exp\left(-i\delta\tau + \frac{1}{2}i\delta^2 z\right)$, $v(z,\tau) \equiv V(z,\tau)\exp\left(+i\delta\tau + \frac{1}{2}i\delta^2 z\right)$, which simultaneously produces an additional Raman term in equations for the new varibles U and V:

$$iU_z + \tfrac{1}{2}U_{\tau\tau} + (|U|^2 + \tfrac{2}{3}|V|^2)U = \epsilon_1\left(|U|^2\right)_\tau U + \epsilon_2\left(|V|^2\right)_\tau U + \epsilon_3\left(UV^*\right)_\tau V - 2i\delta\epsilon_3|V|^2U\,, \quad (4)$$

$$iV_z + \tfrac{1}{2}V_{\tau\tau} + (|V|^2 + \tfrac{2}{3}|U|^2)V = \epsilon_1\left(|V|^2\right)_\tau V + \epsilon_2\left(|U|^2\right)_\tau V + \epsilon_3\left(VU^*\right)_\tau U + 2i\delta\epsilon_3|U|^2V\,. \quad (5)$$

Equations (1) and (2) [or, equivalently, Eqs. (4) and (5)] admit two distinct types of soliton solutions: the *simple* solitons, in which only one component is different from zero, and *vector* solitons, in which $|u| = |v|$. Stability of the solitons in birefringent optical fibers is of obvious interest [5]. The objective of this short paper is to present simple analytical results for the stability of the solitons obtained within the framework of the model based on Eqs. (4) and (5). More approximations will be used below (equal temporal widths of the two components of the vector soliton, and no chirp). However, the main result which will be obtained in this work has a very simple and, simultaneously, general form, which suggests that it may be true beyond the framework of the approximations employed. Namely, it will be demonstrated that the simple solitons are stable provided that the parallel Raman coefficient ϵ_1 is larger than the *net* perpendicular coefficient $\epsilon_2 + \epsilon_3$, while the vector soliton is stable exactly in the opposite case. In virtue of the relation (3), this means that the simple solitons may be stable, while the vector soliton is always unstable. This prediction calls for numerical verification. Numerical simulations of the vector soliton governed by the coupled NLS equations including the full Raman terms were reported in Ref. [4]; however, the system considered in [4] was essentially more complicated, as it included also the linear coupling accounting for the fiber's *twist*. Anyway, a general qualitative conclusion of Ref. [4] was that the Raman effect conspicuously suppressed the effective coupling between the two

*Electronic address: malomed@math.tau.ac.il

polarizations. It seems that the instability of the vector soliton vs the stability of the simple one, predicted in this work, complies with that general conclusion. Analytical consideration of dynamics of the solitons in the presence of the twist-induced linear coupling, acting in combination with the Raman terms, is a more general and challenging problem, which is now under way [6].

In this work, the following simplest ansatz for the soliton's wave form will be adopted:

$$U = \eta \cos\theta \,\mathrm{sech}\,[\eta(\tau - T)] \exp\left[i(-\omega_1\tau + \phi_1)\right] , \quad (6)$$

$$V = \eta \sin\theta \,\mathrm{sech}\,[\eta(\tau - T)] \exp\left[i(-\omega_2\tau + \phi_2)\right] , \quad (7)$$

where η^{-1} is the inverse temporal width of the soliton, which is preassumed to be the same for both components; θ is the *polarization angle*; $\omega_{1,2}$ are the central frequencies of the two components, which, generally, may be different; T is the temporal position of the soliton's center, which is a function of z, but, as well as η, is preassumed to be the same for both components (it is implied that the Raman terms will be regarded as a weak perturbation, while the centers of the two components are kept together by the strong nonlinear cross-phase modulation terms); and $\phi_{1,2}$ are z-dependent phases of the two components.

In the absence of the Raman interactions, the energies of both polarizations

$$W_{1,2} \equiv \int_{-\infty}^{+\infty} |U,V|^2 d\tau \quad (8)$$

are conserved, as well as the net momentum $P \equiv P_1 + P_2$, where

$$P_1 \equiv i\int_{-\infty}^{+\infty} UU_\tau^* d\tau, \quad P_2 \equiv i\int_{-\infty}^{+\infty} VV_\tau^* d\tau . \quad (9)$$

Moreover, if one assumes that $|U(\tau)|^2$ and $|V(\tau)|^2$ are even functions of τ, which is the case in Eqs. (6) and (7), it is easy to check that both components of the momentum, P_1 and P_2, are conserved separately in the absence of the Raman terms.

When the Raman interactions are absent, the conservation laws stipulate that all the essential parameters in the ansatz (6) and (7) are constant: it is straightforward to see that η and θ cannot change, respectively, due to the conservation of $W_1 + W_2$ and $W_1 - W_2$, while ω_1 and ω_2 are conserved if the two momenta P_1 and P_2 are conserved separately. This implies that the coupled NLS equations (4) and (5) should have, at $\epsilon = 0$, exact stationary solutions corresponding to an *arbitrary* polarization θ (i.e., not only to the obvious values $\theta = 0$, $\theta = \pi/4$, and $\theta = \pi/2$). This has been recently demonstrated, in the framework of a more sophisticated analytical approximation, in Ref. [7]; independently, the same result was obtained numerically in Ref. [8].

Inclusion of the Raman terms should give rise to a nontrivial inner dynamics of the soliton represented by the ansatz (6) and (7). A natural assumption is that, if all the Raman terms may be treated as weak perturbations, the ansatz should retain its form, but the parameters θ and $\omega_{1,2}$ will be subject to a slow evolution in z; it is obvious that the inverse width η will suffer no evolution as the net energy remains an exact integral of motion in the presence of the Raman terms. The simplest way to derive the corresponding evolution equations for the parameters of the ansatz is to employ the so-called *balance equations* for the former conserved quantities. For instance, for $W_1 - W_2$ one can immediately obtain from Eqs. (4) and (5)

$$\frac{d}{dz}\int_{-\infty}^{+\infty} \left(|U|^2 - |V|^2\right) d\tau$$
$$= -2\epsilon_3\left[i\int_{-\infty}^{+\infty} (UV^*)_\tau VU^* d\tau + 2\delta\int_{-\infty}^{+\infty} |U|^2|V|^2 d\tau\right] + \text{c.c.} \quad (10)$$

Next, inserting the anstaz (6) and (7) into the left- and right-hand sides of Eq. (10), one arrives at the evolution equation for the polarization angle:

$$\frac{d\theta}{dz} = \tfrac{1}{3}\epsilon_3\eta^2\left[(\omega_1 - \omega_2) + 2\delta\right]\sin(2\theta) . \quad (11)$$

Quite similarly, one can derive the evolution equations for the frequencies $\omega_{1,2}$ from the balance equations for the momenta $P_{1,2}$:

$$\frac{d\omega_1}{dz} = -\tfrac{8}{15}\eta^4\left[\epsilon_1\cos^2\theta + (\epsilon_2 + \epsilon_3)\sin^2\theta\right] , \quad (12)$$

$$\frac{d\omega_2}{dz} = -\tfrac{8}{15}\eta^4\left[\epsilon_1\sin^2\theta + (\epsilon_2 + \epsilon_3)\cos^2\theta\right] . \quad (13)$$

Thus, there are three evolution equations (11), (12), and (13) for the three dynamical parameters θ, ω_1, and ω_2. The simple solitons are represented by two obvious quasistationary solutions of these equations:

$$\theta = 0;\ \frac{d\omega_1}{dz} = -\tfrac{8}{15}\eta^4\epsilon_1;\ \frac{d\omega_2}{dz} = -\tfrac{8}{15}\eta^4(\epsilon_2 + \epsilon_3) , \quad (14)$$

$$\theta = \frac{\pi}{2};\ \frac{d\omega_2}{dz} = -\tfrac{8}{15}\eta^4\epsilon_1;\ \frac{d\omega_1}{dz} = -\tfrac{8}{15}\eta^4(\epsilon_2 + \epsilon_3) . \quad (15)$$

The values of the derivatives $d\omega_1/dz$ and $d\omega_2/dz$ in these solutions exactly correspond to the well-known expression for the rate of the Raman-induced downshift of the soliton's frequency in the model based on the single NLS equation [2]. The frequencies ω_2 and ω_1, respectively, are formal ingredients of the solutions (14) and (15), as, according to Eqs. (6) and (7), the corresponding polarization components are equal to zero in the solutions [these formal terms can be obtained naturally if one considers the simple solitons as a limiting case of general dynamical solutions of Eqs. (11)–(13), corresponding to $\sin(2\theta) \to 0$]. However, the representation of the simple-soliton solutions in the form of Eqs. (14) and (15) is convenient for the subsequent stability analysis.

The vector soliton is given by another quasistationary solution to Eqs. (11)–(13):

$$\theta = \frac{\pi}{4};\ \frac{d\omega_1}{dz} = \frac{d\omega_2}{dz} = -\tfrac{4}{15}\eta^4(\epsilon_1+\epsilon_2+\epsilon_3);$$

$$\omega_1 - \omega_2 = -2\delta\,. \qquad (16)$$

The constant frequency difference -2δ in this solution exactly compensates for the frequency separation introduced when transforming Eqs. (1) and (2) into Eqs. (4) and (5).

To analyze the stability of the solitons, one should linearize the underlying equations (11)–(13) on the background of the corresponding quasistationary solutions. In the case of the simple solitons, linearization of Eqs. (12) and (13) produces trivial equations, while Eq. (11) leads to the following linearized equation for the small perturbation $\theta^{(1)}$ of the polarization angle:

$$\frac{d\theta^{(1)}}{dz} = \pm\tfrac{2}{3}\eta^2\left[\omega_1(z)-\omega_2(z)+2\delta\right]\theta^{(1)}\,, \qquad (17)$$

where the signs + and − correspond, respectively, to the solutions (14) and (15). Inserting into Eqs. (17) the expression for $\omega_1 - \omega_2$ following from Eqs. (14) and (15), one immediately concludes that the simple soliton is stable (i.e., the perturbation $\theta^{(1)}$ does not grow), provided that

$$\epsilon_1 > \epsilon_2 + \epsilon_3\,, \qquad (18)$$

and unstable in the opposite case. Taking into account the relation (3), one immediately concludes that the simple solitons are stable.

To consider the stability of the vector soliton (16), one should combine the linearized version of Eq. (11) for the polarization angle and the linearized equation for $\omega_1-\omega_2$ which can be obtained by subtraction of Eq. (12) from Eq. (13). Assuming that the infinitesimal perturbations of θ and $\omega_1-\omega_2$ are proportional to $\exp(\sigma z)$, a straightforward algebra leads to the following expression for the gain σ:

$$\sigma^2 = \tfrac{16}{45}\epsilon_3\eta^6\left[\epsilon_1-(\epsilon_2+\epsilon_3)\right]\,. \qquad (19)$$

The condition for stability of the vector soliton following from Eq. (19) $\left(\sigma^2<0\right)$ is exactly opposite to Eq. (18): $\epsilon_1 < \epsilon_2+\epsilon_3$, i.e., the vector soliton would be stable if the perpendicular Raman effect were stronger than the parallel one. However, the relation (3) makes this impossible.

Thus, the fundamental restriction (3) imposed by the general physical properties of the nonlinear optical medium [3] leads to the conclusion that the simple solitons may be stable, while the vector solitons may not. Note that otherwise (i.e., if the Raman terms are not included) the vector solitons can be good stable solutions of the coupled NLS equations (see, e.g., Ref. [7]). Although this conclusion is based on the simplest quasi-instantaneous approximation for the Raman terms, as well as on the oversimplified soliton ansatz (6) and (7), it seems plausible that it will remain true in the realistic model with the delayed Raman response and with more freedom given to the solitons.

I am indebted to Yuri Kivshar for valuable discussions.

[1] C.R. Menyuk, IEEE J. Quantum Electron. **QE-25**, 2674 (1989).
[2] J.P. Gordon, Opt. Lett. **11**, 662 (1986); Y. Kodama and A. Hasegawa, IEEE J. Quantum Electron. **QE-23**, 510 (1987).
[3] C.R. Menyuk, M.N. Islam, and J.P. Gordon, Opt. Lett. **16**, 566 (1991).
[4] S. Wabnitz, S. Trillo, E.M. Wright, and G.I. Stegeman, J. Opt. Soc. Am. B **8**, 602 (1991).
[5] K.J. Blow, N.J. Doran, and D. Wood, Opt. Lett. **12**, 202 (1987).
[6] B.A. Malomed and R.S. Tasgal (unpublished).
[7] D.J. Kaup, B.A. Malomed, and R.S. Tasgal, Phys. Rev. E **48**, 3049 (1993).
[8] M. Haelterman, A.P. Sheppard, and A.W. Snyder, Opt. Commun. **103**, 145 (1993).